

An equation of state for unsaturated soils

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Abstract: The enthalpy within a soil system under equilibrium conditions is investigated theoretically. Terzaghi's effective stress equation for a saturated soil is first examined and the approach extended to determine the enthalpy associated with the air, water, and solid phases, and the interactions between the phases, in an unsaturated soil. An equation of state is developed which links the stress state variables ($p - u_a$) and ($u_a - u_w$) to the specific volume (v) and specific water volume (v_w) and thus to the volumes of the phases. The equation is shown to provide a logical interpretation of the average volumetric "coupling" stress p'_c within unsaturated soils which highlights the significance of the dual stress regime and bimodal structure. The new equation is compared with previously reported experimental data on kaolin and a lateritic gravel. The agreement is good and it provides insight into unsaturated soil strength and deformation behaviour and clarifies various previously identified anomalies.

Key words: partial saturation, unsaturated soils, thermodynamics, effective stress, equation of state, critical state.

Résumé : On étudie théoriquement l'enthalpie à l'intérieur d'un système de sol dans des conditions d'équilibre. On examine tout d'abord l'équation de contrainte effective de Terzaghi pour un sol saturé, et son approche élargie pour déterminer l'enthalpie associée à l'air, l'eau et les phases solides, de même que les interactions entre les phases dans les sols non saturés. Une équation d'état est développée qui relie les variables de contraintes d'état ($p - u_a$) et ($u_a - u_w$) au volume spécifique (v) et au volume spécifique de l'eau (v_w), et donc aux volumes des phases. On montre que l'équation fournit une interprétation logique de la contrainte volumétrique moyenne de couplage p'_c dans les sols non saturés qui met en lumière la signification du régime de contraintes doubles et de la structure bimodale. La nouvelle équation est comparée aux données expérimentales déjà publiées sur le kaolin et sur un gravier latéritique. La concordance est bonne et fournit des éclaircissements sur le comportement de la résistance au cisaillement et de la déformation du sol non saturé, et clarifie diverses anomalies qui ont été identifiées antérieurement.

Mots clés : saturation partielle, sols non saturés, thermodynamique, contrainte effective, équation d'état, état critique.

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Introduction

The prediction of soil behaviour is intrinsically linked to the need to determine the controlling stresses in the soil. For saturated soils, Terzaghi (1936) proposed an equation for effective stress which controls the shear resistance and volume changes. The effective stress can be written as

$$[1] \quad p' = p - u_w$$

where

p' is Terzaghi's mean effective stress;

p is the mean total stress; and

u_w is the pore-water pressure.

The concept of the stress state variable ($p - u_w$) as controlling the behaviour of saturated soils has proven very useful and has been shown to be valid in practice. For unsaturated soils, however, the search for a reliable stress state variable equation, independent of soil properties, has proven unsuccessful. As described by Fredlund and

Rahardjo (1993), a number of such equations have been proposed. The original suggestion of Bishop (1959) can be written as

$$[2] \quad p'_B = (p - u_a) + \chi(u_a - u_w)$$

where

p'_B is Bishop's mean stress;

u_a is the pore-air pressure; and

χ is an empirical parameter.

A major obstacle to the use of eq. [2] lies with the parameter χ . This is usually ascribed the range of values $0 \leq \chi \leq 1$ and has been shown to be dependent on the stress path and the process to which the soil is subjected (Jennings and Burland 1962; Blight 1965; Morgenstern 1979; and others).

Although it is desirable that the concept of effective stress for saturated soils is extended to unsaturated soils and that soil properties such as the volumes of the various phases (solid particles, water, and air) are not included in any formulation of controlling stresses, experiments have demonstrated the inadequacy of any such relationship. For this reason, researchers have turned to examining the use of the independent stress state variables ($p - u_a$), ($p - u_w$), and ($u_a - u_w$) to describe the mechanical behaviour of soils. Fredlund and Morgenstern (1977) concluded from theoretical considerations that any two of these three stress state variables can be used to describe the behaviour of an unsaturated soil. However, there are inconsistencies in experimental results

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not readily answered by constitutive modelling using these parameters (Wheeler and Sivakumar 1995). A logical interpretation of experimental data is essential to an appreciation of soil behaviour, and a clear picture does not always emerge using independent stress state parameters, as these interact in response to external stimuli. In this respect, it appears that the volumes of the phases play an important role in controlling the stresses in unsaturated soils, and this is demonstrated in the analysis and the comparisons with both consolidation data and critical state data which follow.

The paper examines the significance of the volumes of the phases, and the interactions between the phases, on the stress regime under equilibrium conditions. First, a description of the significance of enthalpy in soils relating pressures, volumes, and internal energy sources is presented, followed by an examination of Terzaghi's effective stress equation in terms of the enthalpy of a saturated system. This approach is then extended to unsaturated soils to develop an equation of state that includes the average volumetric "coupling" stress p'_c . This links the stress state variables and the volumes of the phases. Lastly, p'_c is compared with published experimental data obtained on both compacted kaolin and a lateritic gravel.

Enthalpy of a soil system

The principles of thermodynamics are of general applicability to the behaviour of gases, liquids, and solids and allow examination of the interactions within a system comprising different phases (e.g., Modell and Reid 1983). Figure 1 presents an idealized compression plot of imposed mean stress p versus total volume V for a soil. Equilibrium or quasi-static equilibrium is assumed at B. The product pV given by the area OABC is taken as the total enthalpy (H_t) of the soil system but needs the datum for p to be established. Though absolute enthalpy is indeterminate, it is sufficient in the analysis to relate pressures to a datum of atmospheric pressure. Enthalpy is a term that defines the combination of certain thermodynamic parameters and for a multiphase continuum has particular significance in that the total enthalpy is the sum of the enthalpies of the phases. Internal to a soil system, the enthalpy of a phase is given by H_n , ($n = 1, 2, \dots$), defined by

$$[3] \quad H_n = p_n V_n + U_n$$

where, adopting the nomenclature used in this paper,

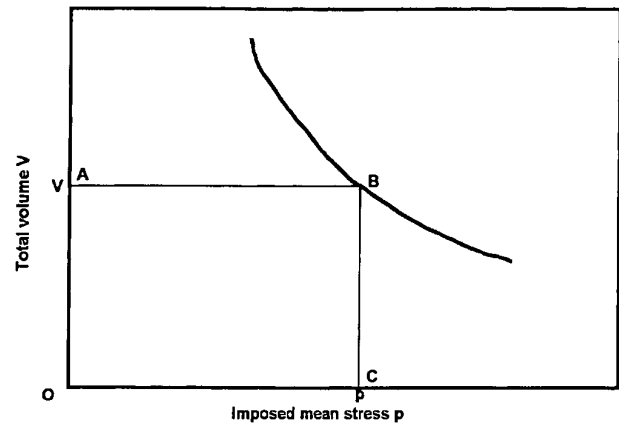
p_n is the pressure in the phase;

V_n is the volume of the phase; and

U_n is the internal energy associated with the phase interactions.

It is the sum of the $p_n V_n$ products of the phases and the internal energies of the interactions between the phases that are taken as equating to the total enthalpy of the soil system. There is no restriction on the path followed in reaching B in Fig. 1, but the enthalpies of the phases are path-dependent, although the sum must equate to the total enthalpy. Besides the phases of a soil comprising the soil particles, water, and air, Fredlund and Morgenstern (1977) proposed that the con-

Fig. 1. Volumetric compression plot for a soil.



tractile skin (the air-water interface) should be considered as a further independent phase. However, the energy associated with the contractile skin is the result of the interaction between the water and the air and would not exist if one of these was not present. For this reason, the contractile skin is treated herein as an interaction between phases rather than as a separate phase. Following a similar argument, the vapour pressure above the contractile skin, the adsorbed water attached to the soil particles, the dissolved air, and the stresses between the soil particles are also treated as interactions in the following.

Terzaghi's effective stress for saturated soils

In accordance with eq. [1], the resistance to total stress p applied to a saturated soil system is provided by the sum of the water pressure u_w and the effective stress p' , where p' is the result of the solid phase being present as discrete particles rather than as a coherent mass. By definition, both p and p' are based on average conditions and in accordance with eq. [3] may be viewed as components of enthalpy per unit total volume of the soil system. The pore-water pressure u_w , on the other hand, acts through the water phase V_w but, because it surrounds the soil particles, also acts through the solid phase V_s . Thus Terzaghi's equation can be written in terms of enthalpy as

$$[4a] \quad pV = u_w V_w + u_w V_s + p'V$$

or

$$[4b]^1 \quad H_t = H_w + H_s$$

where

$$H_t = pV;$$

$$H_w \text{ is the enthalpy of the water phase } (= u_w V_w); \text{ and}$$

$$H_s \text{ is the enthalpy of the solid phase } (= u_w V_s + p'V).$$

As $V = V_w + V_s$ for a saturated soil, eq. [4a] reduces to eq. [1] and the volumes of the phases are eliminated. It should also be noted that no terms for the components of enthalpy associated with the adsorbed water or dissolved air

¹For a saturated soil system under unconfined conditions, $H_t = 0$ and $p' = -u_w$ when atmospheric pressure is taken as the datum. Thus the total and "applied" enthalpies are synonymous.

are present. These interactions are discussed in the following and are shown not to influence eq. [4a].

Enthalpy of unsaturated soils

A generalized equation of state based on the enthalpy of an unsaturated soil system and including the coupling stress p'_c can be written as

$$[5a] \quad pV = u_w V_w + W + C + A + u_a V_a + D + u_s V_s + p'_c V$$

or

$$[5b] \quad H_t = H_w + H_a + H_s$$

where

H_w is the enthalpy of the water phase ($= u_w V_w + W + C + A$);

H_a is the enthalpy of the air phase ($= u_a V_a + D$);

H_s is the enthalpy of the solid phase ($= u_s V_s + p'_c V$);

V_a is the volume of the air phase;

u_s is the stress in the soil particles as a result of u_w and u_a ; and

W , C , A , and D are the internal enthalpy components associated with the water vapour, contractile skin, adsorbed water, and dissolved air, respectively.

The coupling stress p'_c is shown in the following to link the independent stress state variables and gives the average volumetric stress between particles within and between the saturated regions, allowing the behaviour of an unsaturated soil to be more clearly appreciated. In accordance with previous arguments, p'_c can be viewed as a component of enthalpy per unit total volume. In the following, the individual components making up eq. [5a] are discussed and analysed to develop a general form for the equation of state.

Water vapour

Vapour movement is an important means of transport of water within an unsaturated soil and assists in the equalization of water pressures. Under equilibrium conditions within a closed system the vapour pressure is the saturated vapour pressure in air above the contractile skin and is balanced by the water pressure. This is the case for air bubbles under equilibrium conditions in unsaturated soils. Where the air passages are connected to the atmosphere, however, the vapour pressure will be less than the saturated vapour pressure and a true equilibrium may not be established.

The partial pressures of the air and water vapour are defined as the pressures they would exert if each alone occupied the volume of the mixture.² Dalton's divisional law states that the total pressure is equal to the sum of the partial pressures of the constituents. From eq. [5a], this is consistent with

$$[6] \quad u_a V_a + W = (u_a + u_v) V_a$$

where

u_v is the water vapour pressure; and

V_a is the volume of air and is taken to include the volume of water vapour.

Schuurman (1966), in discussing the role of air bubbles in water, states that the compressibility of air bubbles is not influenced directly by the saturated vapour pressure; there is only an influence on u_w . Schuurman further shows that the influence on u_w is small and the influence of the water vapour decreases as u_a increases, as for smaller and smaller air bubbles. At 20°C the vapour pressure above a flat water surface, which thus represents likely worst conditions, is around 2.4 kPa and the volume percentage of water vapour in air is likely to be less than 1%. Thus the contribution of the water vapour to the total enthalpy of a soil system is small and W in eq. [5a] can be disregarded.

Dissolved air

In general, the change in the volume of air within a soil may be a result of (i) the interchange of air with the external surroundings, (ii) compression or expansion of the free air in the pores, or (iii) air dissolving or being liberated from the pore water. At equilibrium, when water is in contact with free air, the volume of air dissolved in the water is essentially independent of air and water pressures, as can be demonstrated by examination of the ideal gas laws and Henry's law (Fredlund and Rahardjo 1993). It is the liberation of the dissolved air under decreasing external pressure and corresponding increasing suction which leads to the formation of air bubbles. As shown by Dorsey (1940), however, dissolved air produces no significant difference between the compressibility of deaired water and air-saturated water. Additionally, the volume of dissolved air, approximately 2% of the volume of water, does not influence the total volume of the soil system, as it is incorporated within the water without any significant change in volume. This suggests that the pressure of the air dissolved in water has little effect on the pore-water pressure. Thus the contribution of the component of enthalpy D in eq. [5a] due to dissolved air, to the total enthalpy of a soil system can be ignored.

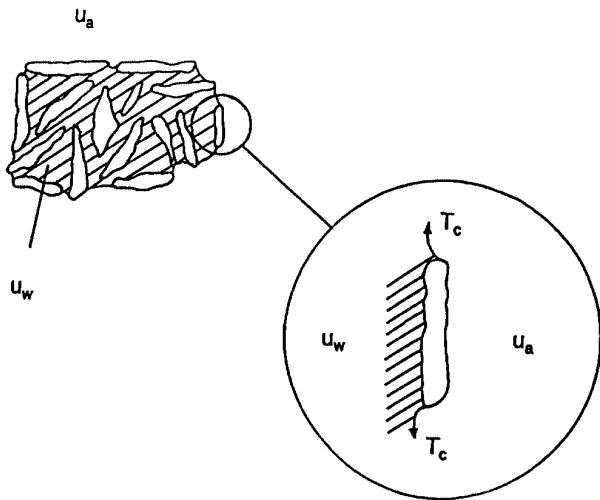
Solid particles

The effective stress p' in saturated soils is treated separately to other stresses within the soil particles in appraising the enthalpy of a soil system. While the soil remains saturated, or at high degrees of saturation, the particles are wholly surrounded by water and the enthalpy component due to water pressure is given by $u_w V_s$ as in Terzaghi's eq. [4a]. Similarly, the coupling stress p'_c in unsaturated soils is designated as a distinct and separate stress to other stresses acting within the soil particles as a result of the influence of air and water pressures.

Barden and Sides (1970) describe unsaturated soils as comprising saturated packets surrounded by air-filled voids (see Fig. 2). This is considered a useful idealization in developing the analysis and in visualizing the bimodal structure considered to exist in unsaturated soils, with different stress regimes within and between the saturated regions. The concept is not considered to limit the analysis or the conclusions reached, although only at low degrees of saturation is the water phase not likely to be continuous. Within the idealized packets the particles will be closely packed and the saturated

²This is analogous to the condition in a saturated soil where the water pressure of eq. [4a] acts through the total volume of the soil comprising the water phase and the solid phase.

Fig. 2. Saturated packet.



packets are taken as comprising the combined volumes of the solid and water phases. The pressure u_s in eq. [5a] contributing to the enthalpy of the particles is likely to vary depending on whether the particles are on the periphery or internal to the packets. The water pressure u_w can be taken as acting on that part of the surface area of the particles internal to the packets, and u_a can be perceived as acting on the remaining surface area of the particles, with the surface tension required to maintain continuity between the pressures. The pressure in the particles is likely to be close to u_w , or only slightly greater, as will be demonstrated by analysis of the results of Wheeler and Sivakumar (1995) in Appendix B. However, a general expression for the component of enthalpy due to air and water pressures is given by

$$[7] \quad u_s V_s = a u_w V_s$$

where the dimensionless parameter a has a minimum value of 1 as $u_a \geq u_w$.

Contractile skin

A manifestation of the energy within the contractile skin is a surface-tension effect that acts within a layer a few molecules thick. As illustrated in Fig. 3, in three dimensions, equilibrium of the contractile skin is given by Kelvin's equation (with the omission of a term for the vapour pressure which, as previously argued, is small and can be neglected):

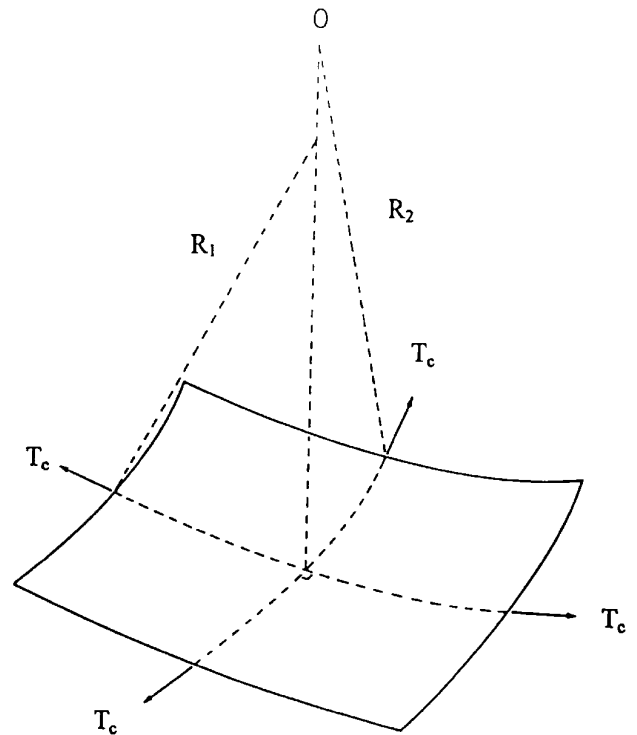
$$[8] \quad u_a - u_w = T_c \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

where

- T_c is the surface tension in the contractile skin with units of force per unit length; and
- R_1 and R_2 are the radii of curvature of the contractile skin for two perpendicular planes at right angles to the curvature of the interface.

As shown by Childs (1969), the contractile skin may be wholly concave or a combination of concave and convex (in two orthogonal directions) to the higher pressure air. It is only necessary that the relative magnitudes of R_1 and R_2 are

Fig. 3. Surface tension in the contractile skin.



such as to balance eq. [8]. Indeed, where the contractile skin spans between collections of soil particles, dependent on the geometry, it is necessary for continuity that the interface has curvature both concave and convex to the air phase.

For the case of spherical occluded air bubbles of radius R ($R = R_1 = R_2$ in eq. [8]), the component of enthalpy of the contractile skin at equilibrium is given by the surface tension multiplied by the total surface area of the bubbles. This assumes that the volume of water associated with the contractile skin is included in the volume of the water phase. Thus, for relatively uniform size spherical bubbles, it is readily shown that the component of enthalpy associated with the contractile skin as a result of surface tension is given by the following equation:

$$[9] \quad \frac{3}{2} V_a (u_a - u_w)$$

Careful consideration needs to be given to the physics of air bubbles to appreciate their significance in the transition from a saturated soil to a soil comprising saturated packets surrounded by air-filled voids. Keller (1964) discusses the growth and decay of gas bubbles in liquids and suggests a dynamic system based around Kelvin's equation. There is likely to be a relatively abrupt energy change in the formation of air bubbles and the development of a contractile skin. A bubble in the pore water may decay in size as the gas dissolves or grow if gas is liberated from solution into the bubble. In discussing the general framework of bubble growth and decay it is important to distinguish between the different degrees of air saturation in water. In the saturated condition the free air pressure is taken as equal to the pressure of the dissolved air, whereas in a supersaturated solution the dis-

solved air pressure exceeds the free air pressure and in an undersaturated solution the dissolved air pressure is less than the free air pressure.

Consider the situation under confined conditions where water is initially in an air-saturated condition. Though bubbles may form spontaneously around nuclei such as a solid particle, decreasing the confining pressure will result in the water becoming supersaturated and dissolved air will start to be more readily liberated. There must be a reduction in air pressure in the bubbles and of the dissolved air as air is liberated from solution, but the pressure inside the bubbles must exceed the water pressure outside the bubbles. For spherical bubbles, there is an equilibrium radius R_e based on eq. [8] which will be dictated by the equilibrium of the pressure of the dissolved air and the air in the bubbles. This represents an unstable equilibrium and any bubbles with a radius smaller than R_e will tend to decay, as the pressure inside the bubbles will be greater than the dissolved air pressure, and any bubbles with a radius greater than R_e will tend to expand, as the pressure will be too small.

In a supersaturated closed water system with no soil particles, it is necessary for continuity to conjecture the tendency for the free air to form in a single bubble that gradually approaches equilibrium. This bubble is found at the top of a container of water. In support of this, Keller (1964) points to the observation of the Russian astronaut Colonel Nikolayev, who while orbiting around the world in a satellite reported that in a closed bottle containing liquid and gas "the gas formed a single spherical bubble near the centre of the bottle." For a water-air solution under reduced confining pressure, where there is no influence from the sides of the container, the dissolved air pressure reduces more rapidly than the free air pressure and equilibrium (an air-saturated condition) is gradually approached with a single bubble present.

In soils the situation is more complicated because of the presence of soil particles. Under decreasing external pressure, bubbles will expand until they achieve equilibrium within the confines of the pore spaces. Within these spaces the bubbles will interact with the soil particles, with which they are likely to have a close affinity, and they will also be influenced by the adsorbed double layer and gravity, and are unlikely to be spherical. Under decreasing confining pressure these air "pockets" will have a tendency to expand and link with others to form larger pockets until eventually continuous air passages are established.

Under the alternative of increasing confining pressure from an equilibrium condition, the water will be undersaturated, as the free air will have pressures greater than the dissolved air and will tend to dissolve to equalize conditions, although again influenced by the presence of the soil particles.

The rate of expansion and decay of the volume of free air is likely to influence the perceived equilibrium conditions. Barden and Sides (1967) concluded that in unsaturated soils there is evidence that equilibrium in terms of Henry's law may require a considerable time interval, far greater than that in the absence of soil particles.

In the idealized case of occluded spherical air bubbles, the component of enthalpy C in the contractile skin (eq. [5a]) is controlled by the volume of air as suggested by eq. [9].

However, at larger suctions where the air passages are continuous and the pore water collects in the saturated regions, it is considered more appropriate to relate C to the sum of the volumes of the water and solid phases ($V_w + V_s$) (though it is noted that for a given volume of soil V , if V_a is known, then $(V_w + V_s)$ may be determined, and vice versa). Appendix A contains a derivation of the equation for the component of enthalpy of the contractile skin of saturated packets justifying this approach. Noting the form of eq. [9], this leads to the component of enthalpy C associated with the contractile skin:

$$[10] \quad C = b(V_w + V_s)(u_a - u_w)$$

where b is a dimensionless number influenced by the structure and size of the saturated packets ($= 3V_a/2(V_w + V_s)$) for idealized occluded (spherical) air bubbles).

The component of enthalpy given by eq. [10] is small, as demonstrated in Appendix B, where b is shown from experimental results on kaolin to be close to zero. However, the surface tension plays an important role in the hysteresis effects observed during drying and wetting of a soil as the contractile skin either resists pores emptying or filling with water, respectively.

Adsorbed double layer

In clays the particles are generally considered to carry a net negative charge. When the particles are surrounded by sufficient free water this is balanced by the cations in the adsorbed double layer. For isolated particles under these conditions there would be no net charge and no net component of enthalpy due to the surface charge. However, the surface charge comes into play when (i) the double layer of particles come into contact; and (ii) there is a reduction of the cations within the adsorbed layer, as would be expected when air comes into contact with the double layer. In these cases a net repulsive force comes into play. The particles also experience attractive forces, notably van der Waals' force, which affects all adjacent pieces of matter and is essentially independent of the characteristics of the fluid between the particles (Sposito 1989). The enthalpy associated with the volume of adsorbed fluid A in eq. [5a] is considered small and can be ignored, particularly as in part its volume is included in the volume of the water phase. However, the repulsive and attractive forces are of significance and are deemed to be included in the formulation of p'_c , as they are for Terzaghi's effective stress in saturated soils.

Average volumetric coupling stress (p'_c)

The physical interpretation of p' for saturated soils is closely related to the stress (or unit force) transmitted through the particles but is not merely the contact stress between particles (Lambe and Whitman 1969). Indeed, in a highly plastic, saturated, dispersed clay there may be little or no physical contact between the particles. In the equation of state for unsaturated soils, p'_c must incorporate not only the influences of changes in the externally applied stresses but also the influences of changes in the degree of saturation which produce additional changes in the repulsive and attractive forces between the soil particles. Thus, p' and p'_c represent macroviews of the microscopic forces between particles.

As a soil compresses or expands, the particles move closer together or farther apart. The resulting interactions of the double layers influence p'_c . In an unsaturated soil, where there are menisci between particles, there must also be interaction between the contractile skin and the adsorbed double layer and presumably a reduction in the thickness of the double layer. This is unlikely to be a static environment, though a quasi-static equilibrium between surface tension forces, water vapour, and the adsorbed water is likely to be established with time.

With a decrease in moisture content, however, there will be an increase in the repulsion forces between closely spaced particles, which along with the influence of the contractile skin may be perceived as helping to propagate air-filled void spaces, the creation of saturated packets, and aggregation of the soil particles. Thus there is a difference in the concentration of stresses within the saturated packets from those between the saturated packets. As outlined in Appendix A, since under equilibrium conditions u_w will equalize throughout the saturated regions, Terzaghi's effective stress is considered to apply to that volume comprising $(V_w + V_s)$. This helps to distinguish p' from p'_c , which relates to the unsaturated soil as a whole and includes a component for the interaction between the saturated regions.

General equation of state

As shown in the preceding sections, the analysis can be simplified by ignoring the small effects of D , W , and A in eq. [5a] on the total enthalpy of the soil system. If this is done, substituting eqs. [7] and [10] into eq. [5a] leads to the following general equation of state for unsaturated soils:

$$[11] \quad p = u_w n_w + b(n_w + n_s)(u_a - u_w) + u_a n_a + a u_w n_s + p'_c$$

where

- n_a is the air porosity ($= V_a/V$);
- n_w is the water porosity ($= V_w/V$);
- $n_s = V_s/V$; and
- $(n_w + n_s)$ represents the total volume of the saturated packets per unit volume of soil.

Using eq. [11] it is possible to highlight the significance of the stress state variables $(p - u_a)$, $(p - u_w)$, and $(u_a - u_w)$ for unsaturated soils and their implicit relationship with the volumes of the phases. As illustrated in Fig. 4, the plots of u_a and u_w versus p in unsaturated soils are not linear, as is often taken for the saturated state, and u_a has a value greater than u_w . This means that p'_c in the unsaturated state is less than p' would have been if the soil had remained saturated. Rearranging eq. [11], the following form of the equation can be obtained:

$$[12]^3 \quad p'_c = (p - u_a) + \alpha(n_w + n_s)(u_a - u_w)$$

where α is a dimensionless parameter defined as

$$[13] \quad \alpha = 1 - b - (a - 1) \frac{n_s}{n_w + n_s} \frac{u_w}{u_a - u_w}$$

The analysis to determine b in Appendix A adopts as a principle that Terzaghi's effective stress is applicable within the saturated packets and that $(u_a - u_w)$ alone is influential in creating the packets. The sum of the enthalpies of the individual components of the system in isolation, before the phases are combined, is determined and is taken as equal to the enthalpy of the saturated packets after they are combined. The basis of this is that under isothermal conditions, provided the volumes of the water and particles remain unchanged, no work is done on the system.

Substituting for b from eq. [A5] in Appendix A into eq. [13] gives

$$[14] \quad \alpha = 1 + \frac{5}{2} \frac{u_a - u_w^*}{u_a - u_w} \frac{n_w}{n_w + n_s}$$

where $(u_a - u_w^*)$ is negative, being the difference between the external air pressure and the water pressure inside a spherical bubble of water with the same volume as that of the water within a saturated packet (u_w^* is the water pressure in a droplet of radius R^*).

From eq. [14], for a dry soil $n_w = 0$ and $\alpha = 1$, and from eq. [12] for a saturated soil, if Terzaghi's equation holds true, again $\alpha = 1$. Between these two extremes α may have a value < 1 . As shown in Appendix B, however, a comparison with the experimental results of Wheeler and Sivakumar (1995) suggests that taking α as unity provides a reasonable assumption at intermediate degrees of saturation. The error in this assumption is shown to be small and decreases with increasing $(u_a - u_w)$ and with an increase in the size of the saturated packets. A value of $\alpha = 1$ is thus thought to be applicable well beyond the experimental range examined and includes soils where the water phase is continuous. Under reasonably worse conditions with $(u_a - u_w) = 100$ kPa and silt-size saturated packets, α is assessed as being not less than 0.96, whereas for gravel-size saturated packets, under otherwise similar conditions, α is approximately 0.99. On this basis, eq. [12] can be written as

$$[15a] \quad p'_c V = (p - u_a)V + (u_a - u_w)(V_w + V_s)$$

or, in terms of specific volume v and specific water volume v_w (Wheeler 1991),

$$[15b] \quad p'_c = (p - u_a) + s \frac{v_w}{v}$$

or, in dimensionless form,

$$[15c] \quad \frac{p'_c}{s} = \frac{p - u_a}{s} + \frac{v_w}{v}$$

where

- s is the suction ($= u_a - u_w$);
- $v_w/v = (n_w + n_s)$;
- $v = 1 + e$ (e is the voids ratio); and
- $v_w = 1 + e S_r = 1 + w G_s$ (S_r is the degree of saturation, w is the water content, and G_s is the specific gravity of the soil particles).

Equation [15a] is presented graphically in Fig. 5 and shows the makeup of the total enthalpy pV for an idealized

³Equation [12] can be written in terms of the stress state variables $(p - u_w)$ and $(u_a - u_w)$, or $(p - u_a)$ and $(p - u_w)$.

Fig. 4. Responses of u_a and u_w during monotonic undrained unloading.

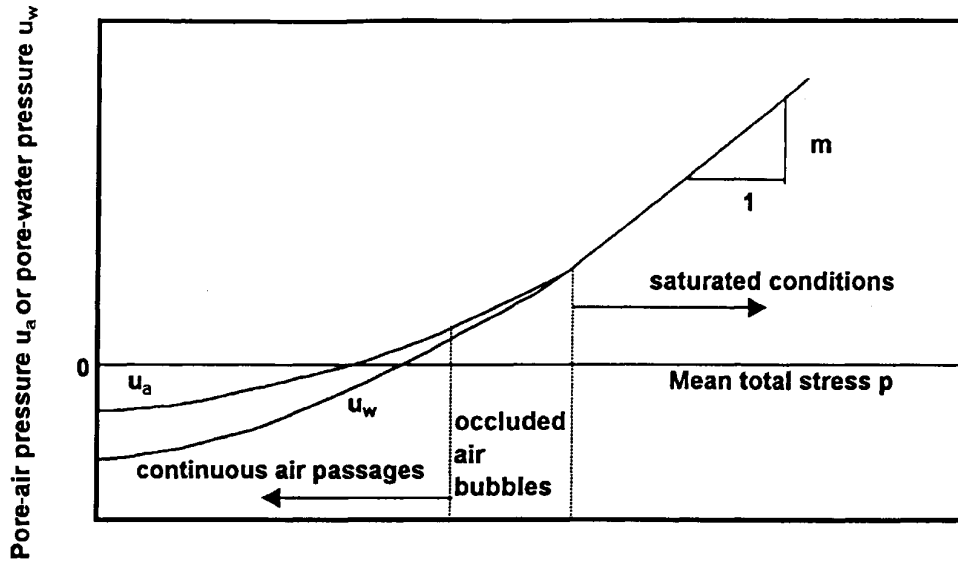
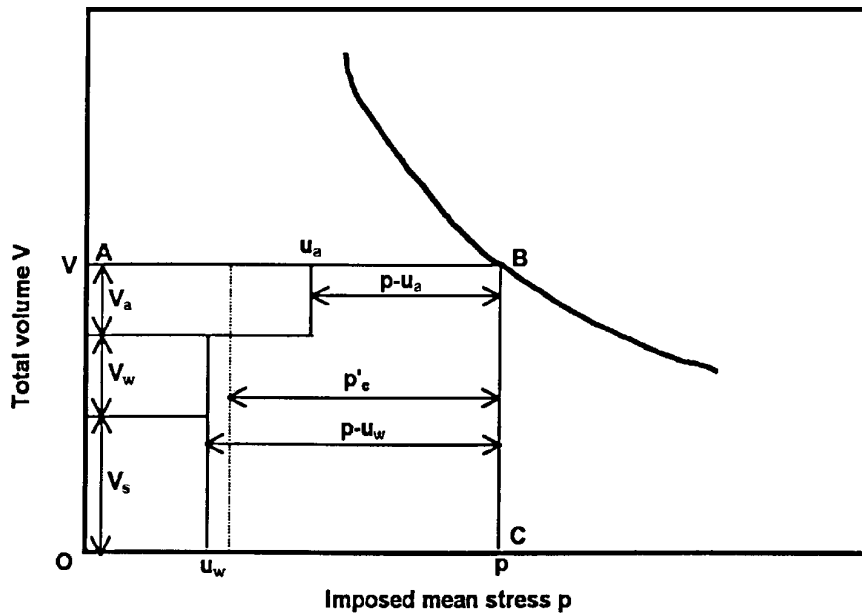


Fig. 5. Idealized volumetric compression plot for an unsaturated soil.



compression plot for an unsaturated soil as in Fig. 1. The pore-water pressure u_w is shown as positive, as is generally the case in triaxial testing, but a similar relationship (eq. [15a]) can be derived for negative values of u_w provided the appropriate sign convention is employed. An alternative form of the equation of state, eq. [15b], can be compared with eq. [2] of Bishop (1959), but the two equations should not be confused. Bishop's equation relates to the assumption of a single controlling stress p'_B . However, the equation of state should be viewed as comprising two components: the stress state variable $(p - u_a)$, which acts throughout the whole soil system with the saturated packets acting as large "particles"; and the stress state variable $s = (u_a - u_w)$, which acts within the saturated regions only. There is thus a greater

"effective" stress level $(p - u_w)$ within the saturated regions. The coupling stress p'_c links the stress state variables with the volumes of the phases.

For a saturated soil, $v_w/v = 1$ and eq. [15b] reduces to Terzaghi's equation as given by eq. [1]. For an unsaturated soil, v_w/v is the volume of the saturated packets per unit volume of soil. As the soil dries, v_w/v approaches $1/v$ and p'_c reduces to $(p - u_a) + s/v$, with the second term being a result of the high suction arising from the very small water volume remaining where particles are closest together. Only on the complete breakdown of the suction does $p'_c = (p - u_a)$. A reduction in the suction at packet-particle contacts would be necessary to facilitate collapse of a soil resulting in a more compact equilibrium state. Internal localized shearing is an

essential feature of collapse and requires that there are sufficient air voids between the saturated packets and a detrimental change in the stress regime due to the addition of water. In this respect it is important to distinguish between externally applied stresses and those internal to the soil system, as these are not in balance during collapse. Although external stresses may remain constant, the addition of water results in a reduction in s , thus in p'_c , with a consequential reduction in the strength of the soil compatible with collapse. The reduction in s locally (particularly between the agglomerated packets) also means that shear stresses are set up. Loose and normally consolidated soils are likely to be more prone to yielding and exhibiting a collapse mechanism. Static equilibrium in unsaturated soils can be viewed as unstable if kinematic energy, leading to collapse, can be released by the addition of water.

Comparison with published experimental results

Figures 6–13 have been prepared based on the experimental data reported by Wheeler and Sivakumar (1995) and Toll (1990). In the following analysis, the prime difference between the approach used in this study and those of Wheeler and Sivakumar and Toll is that the coupling stress p'_c of eq. [15] has been used in place of the mean net stress ($p - u_a$). However, it should be noted that p'_c is not a replacement for the two stress state variables, and the suction component is still needed as the second variable. For brevity, full details of the test procedures are not presented. The results of Wheeler and Sivakumar comprise several series of triaxial tests under different loading conditions and at four different suction values s of 0, 100, 200, and 300 kPa on compacted kaolin clay. The suctions were maintained constant during the tests using the principle of axis translation. The results of Toll comprise triaxial shearing tests with suctions between 0 and 537 kPa on a lateritic gravel. The water contents were kept constant during the tests. The analysis thus examines widely differing materials under different test conditions.

Consolidation

Figure 6 presents results from the isotropic consolidation stages of the triaxial tests of Wheeler and Sivakumar (1995). The tests were carried out under different confining pressures. Figure 6 indicates consistent reproducible results and a clear linearization of the plots of specific volume v versus p'_c as the consolidation pressure is increased. The straight-line portions of the plots can be defined by

$$[16] \quad v = N_t - \lambda_t \ln p'_c = N_t - \lambda_t \ln \left[(p - u_a) + s \frac{v_w}{v} \right]$$

where

eq. [15b] has been used to substitute for p'_c ;

N_t is the extrapolated value of v at $p'_c (=1.0 \text{ kPa})$; and

λ_t is the slope of the straight-line portion of the plots.

The values of N_t and λ_t plotted in Fig. 7 increase with increasing s , consistent with the compression lines for unsaturated soils converging on the compression line for a saturated soil. This is considered logically correct, since for constant s the relative influence of $s(v_w/v)$ in eq. [16] would decrease as $(p - u_a)$ increased. Using p'_c clarifies the signifi-

cance of the divergence of the consolidation lines for increasing s when using $(p - u_a)$ as suggested by Alonso et al. (1990). Normalizing p'_c with respect to s and plotting versus v_w/v , a relationship suggested by eq. [15c], Fig. 8 indicates the consolidation data as tending towards an "envelope of isotropic consolidation," with the deviance possibly being explained by the "built-in" anisotropic compaction stresses within the specimens.

Shearing at the critical state

Figure 9 presents results from the shearing stages of the triaxial tests of Wheeler and Sivakumar (1995). The deviator stress q has been plotted against p'_c at the critical states. All lines are tentatively interpreted as linear and parallel within the range of data presented. The question must be asked is why do the plots for greater suctions lie below the line for $s = 0$? One of the reasons is that within an unsaturated soil, there is likely to be preferential shearing between the saturated packets where stress levels are lower than the average. Normalizing both q and p'_c with respect to s , however, Fig. 10a suggests the results for kaolin lie close to a unique line. Similarly, Fig. 10b based on a reappraisal of the results of Toll (1990) shows a distinct linear relationship for all suctions. The only results not included in Figs. 10a and 10b are those where $s = 0$ or is very close to zero, a situation that is discussed below. On this basis, the following relationship can be written:

$$[17] \quad \frac{q}{s} = M_t \left(\frac{p'_c}{s} - 1 \right) + \Lambda$$

where

M_t is the total stress ratio; and

Λ is the intercept on the q/s axis at $p'_c/s = 1$.

Substituting for p'_c from eq. [15b] gives

$$[18] \quad q = M_t(p - u_a) + sM_s$$

where M_s is the suction stress ratio ($= [M_t(v_w/v - 1) + \Lambda]$).

Equation [18] is in a form similar to that proposed by Toll (1990) and includes M_t and M_s for critical state conditions, the latter stress ratio being shown to be a function of v_w/v . Of note in Figs. 10a and 10b is that $\Lambda = 0.6$ appears to be true for both the tests on kaolin and those on gravel. The significance of this finding is uncertain but at $p'_c/s = 1$ the average volumetric stress p'_c would be s and the corresponding q would equal $0.6s$, suggesting that the deviator stress is a distinct function of p'_c . This may relate to the shape of the yield envelope under these specific conditions.

It is important also to consider the case of $s = 0$ (i.e., $u_a = u_w$) when assessing the significance of the stress ratios. The critical state line for saturated soils is given by $q = p'M$, where M is the critical state stress ratio, but it is not correct to take $M_t = M$ as suggested by eq. [18]. This equality would only hold true if the lines of Figs. 10a and 10b were to pass through the intercept $q/s = p'_c/s = 0$, in which case Λ would equal M . This assumption is made by Toll (1990) in his determination, by multiple regression techniques, of the critical state stress ratios. As the experimental evidence indicates, $\Lambda < M$ for both kaolin and gravel, and the value of M_t , though constant, is greater than M . From the results of Wheeler and Sivakumar (1995) for kaolin, $M = 0.82$ and

Fig. 6. Variation of specific volume during ramped consolidation.

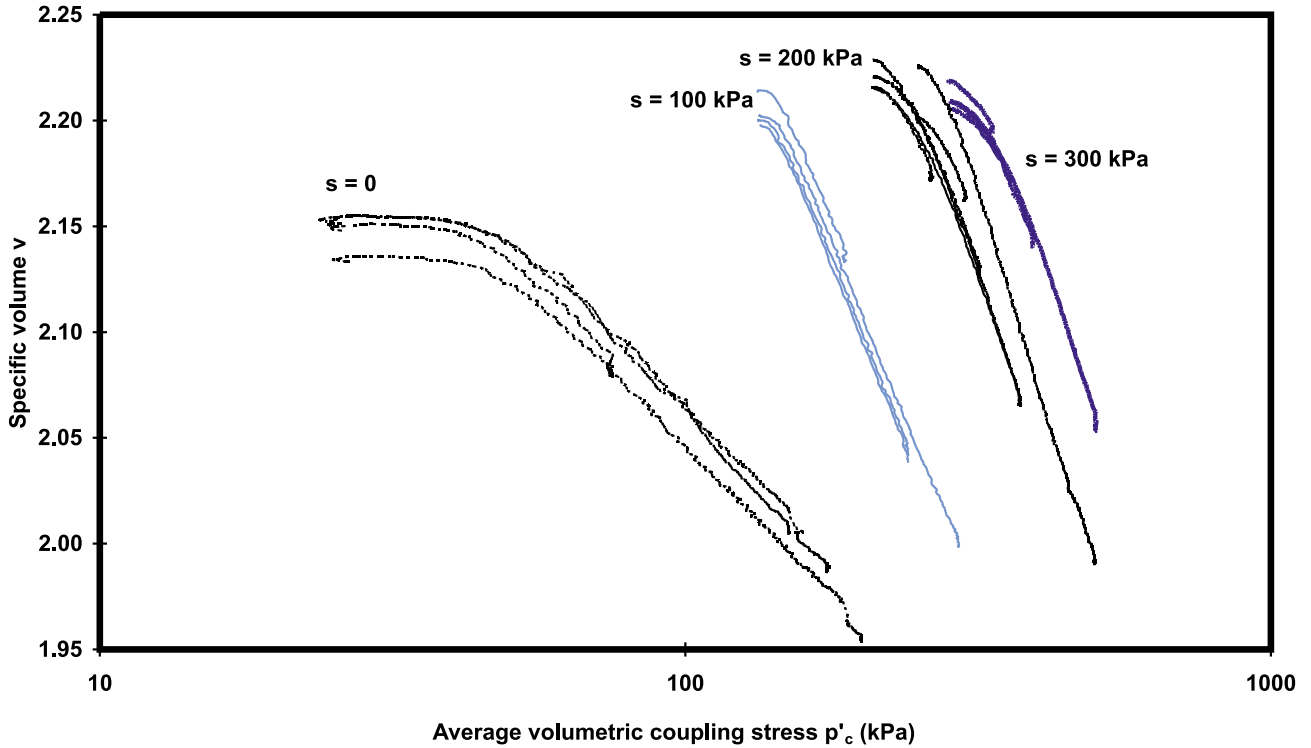
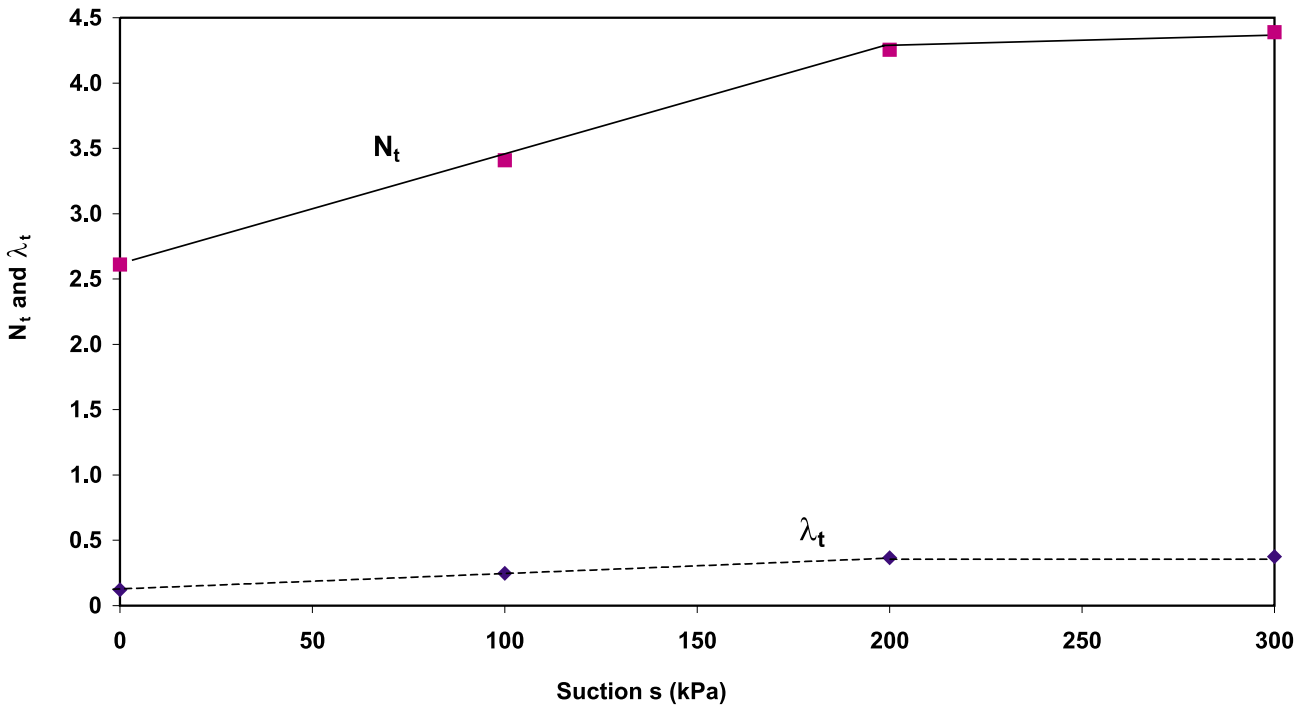


Fig. 7. Plots of N_t and λ_t versus suction s from the consolidation data of Wheeler and Sivakumar (1995).



$M_t = 0.86$. The results of Toll for gravel suggest $M = 1.62$ and $M_t = 1.76$. Though the differences are not large, $M_t > M$ is interpreted as implying that the creation of saturated regions or packets results in the agglomeration of particles

which act as “larger particles” (Toll 1990). Once created, however, the modified soil structure results in a constant M_t . The component of shearing resistance $M_t(p - u_a)$ in eq. [18], which relates to the volume of the soil as a whole, suggests

Fig. 8. Plots of v_w/v versus normalized q/s at critical states and under isotropic consolidation.

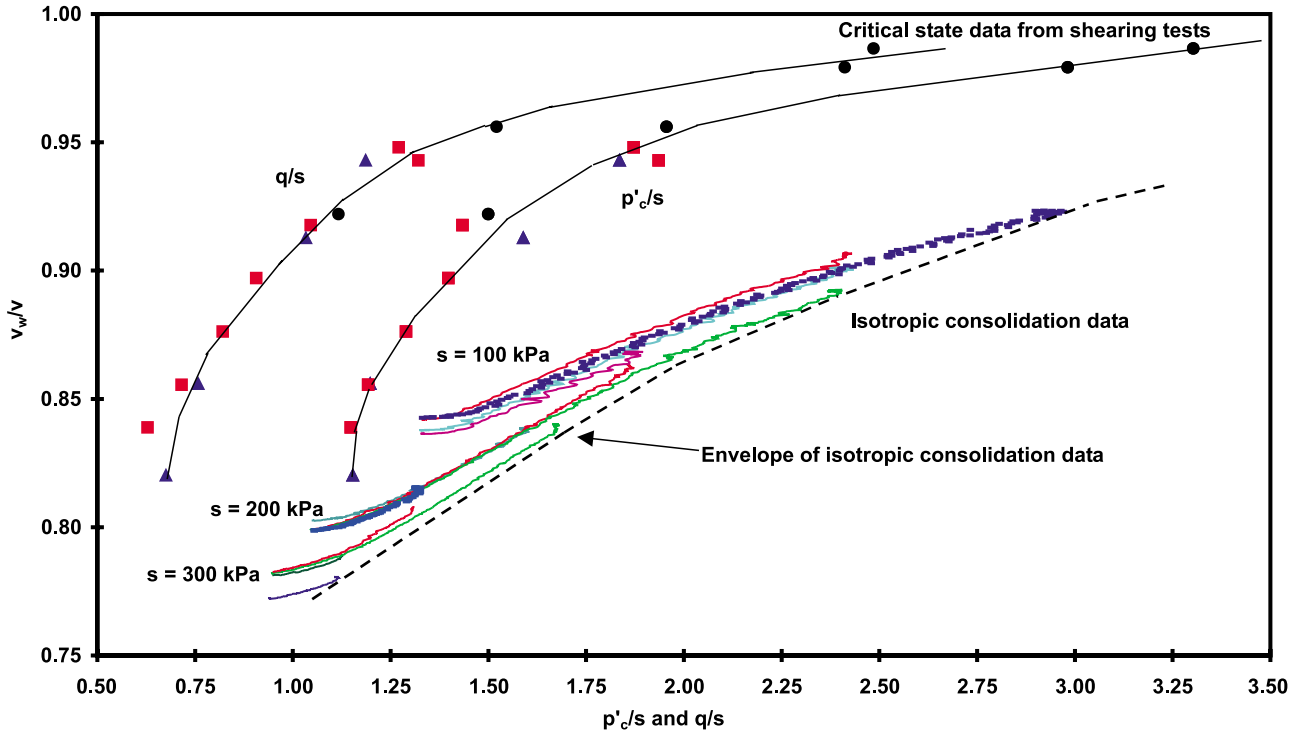


Fig. 9. Plots of q versus p'_c at critical states from the data of Wheeler and Sivakumar (1995).

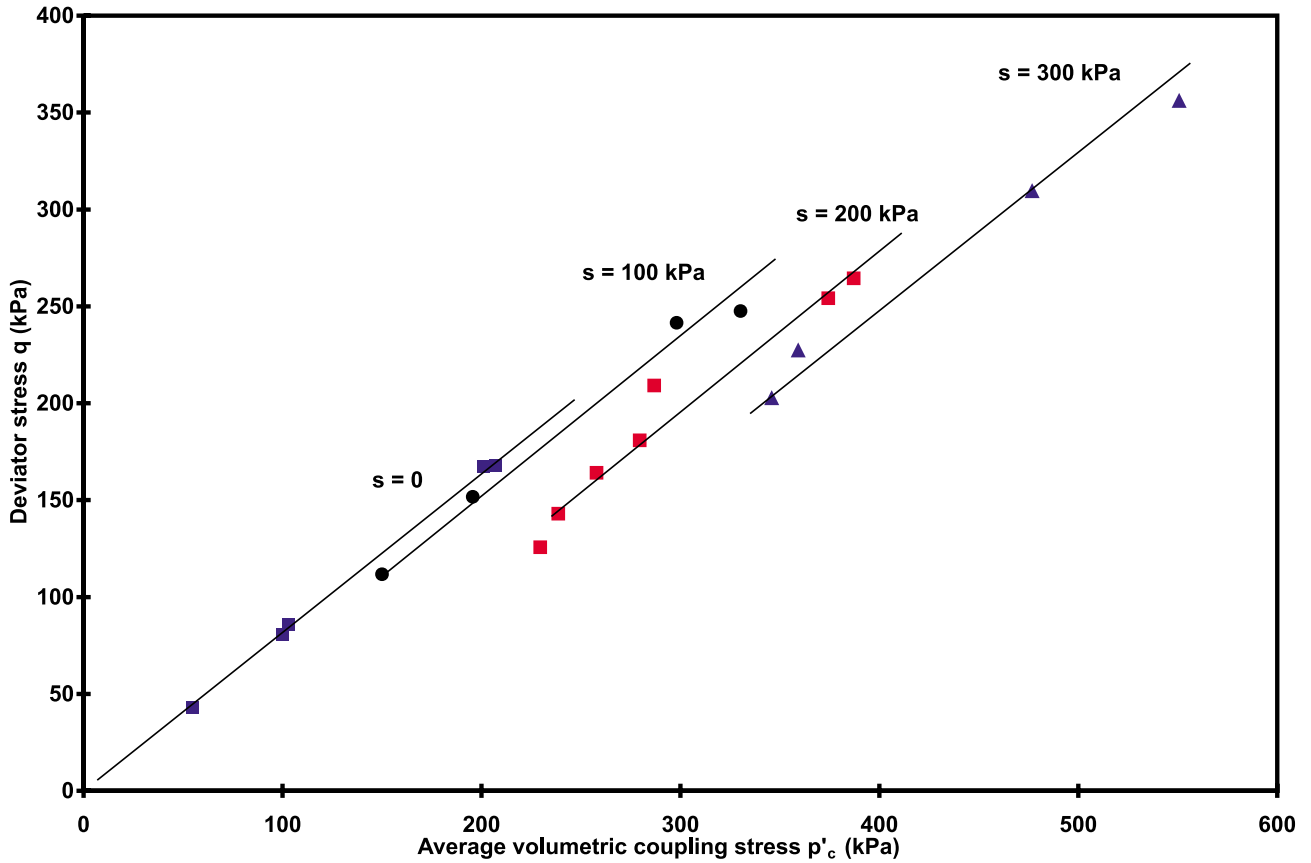


Fig. 10. Plots of q/s versus p'_c/s at critical states from the results of (a) Wheeler and Sivakumar (1995), and (b) Toll (1990).

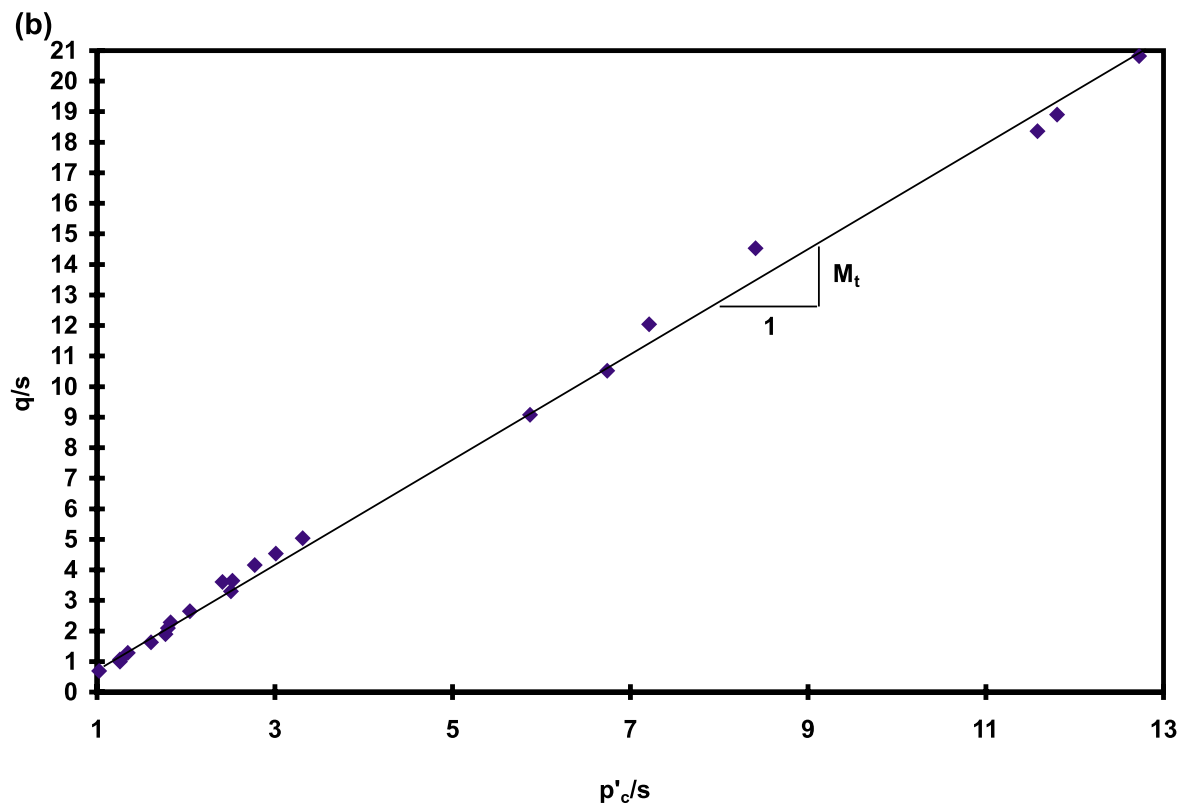
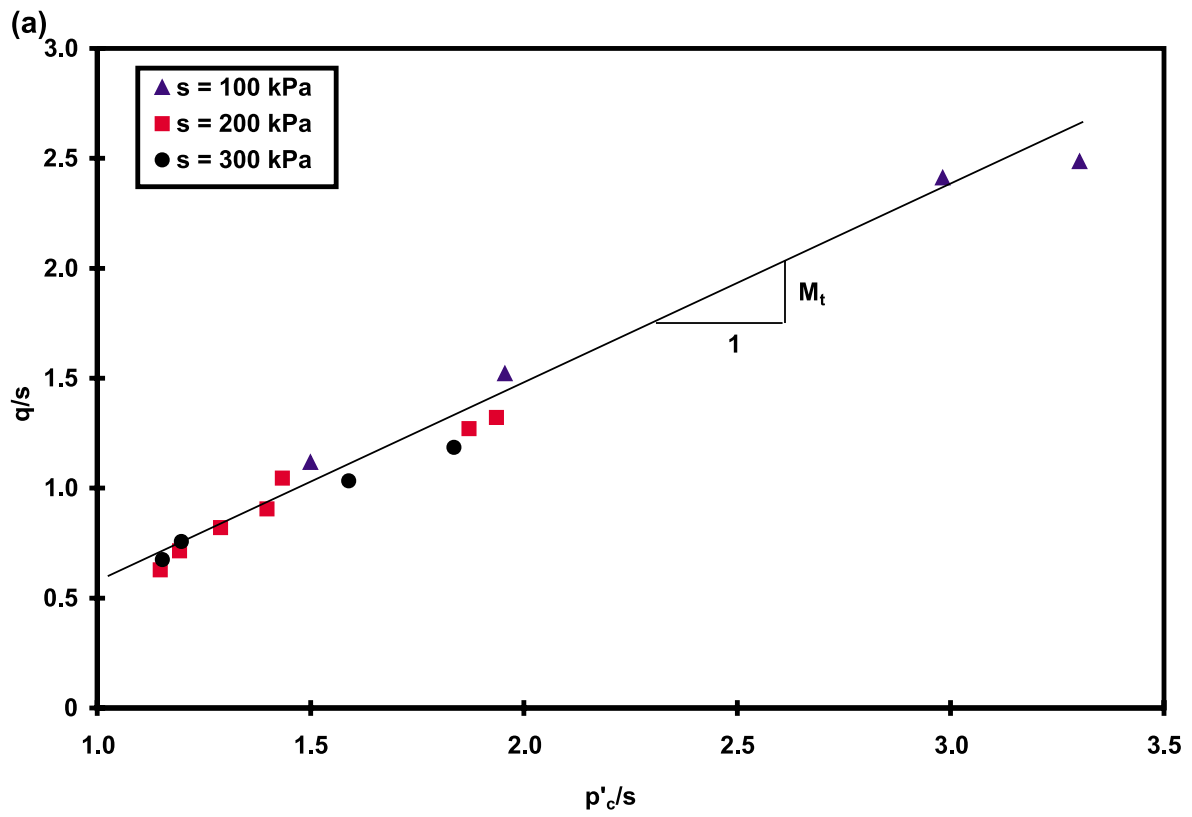


Fig. 11. Plots of M_s versus v_w/v from the experimental results of Wheeler and Sivakumar (1995) and Toll (1990).

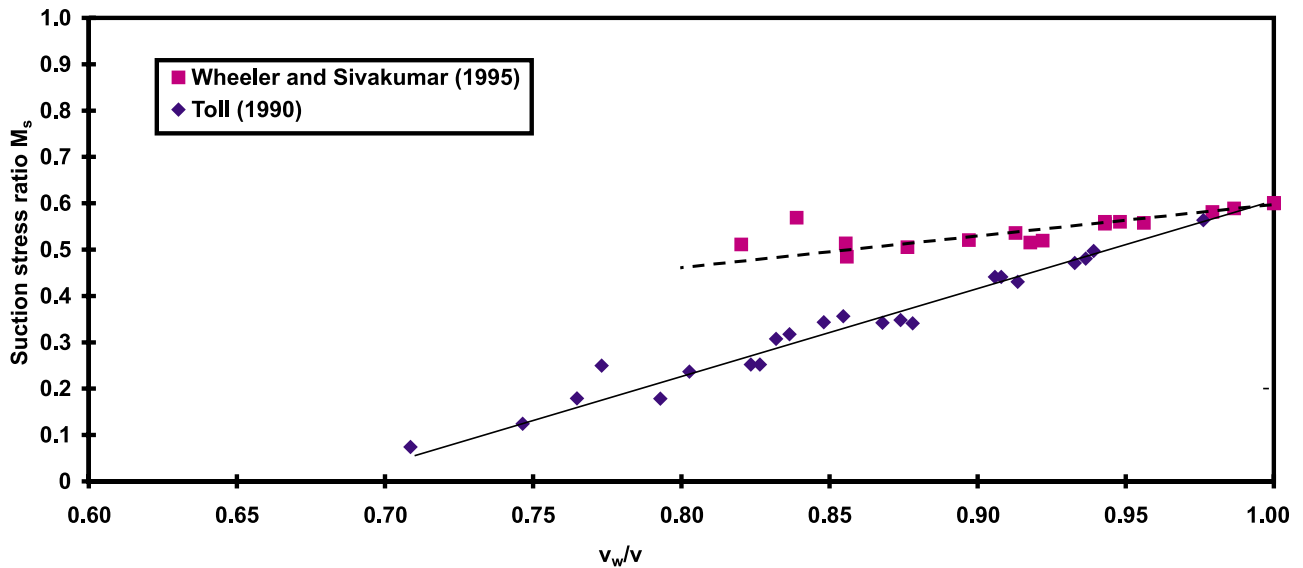
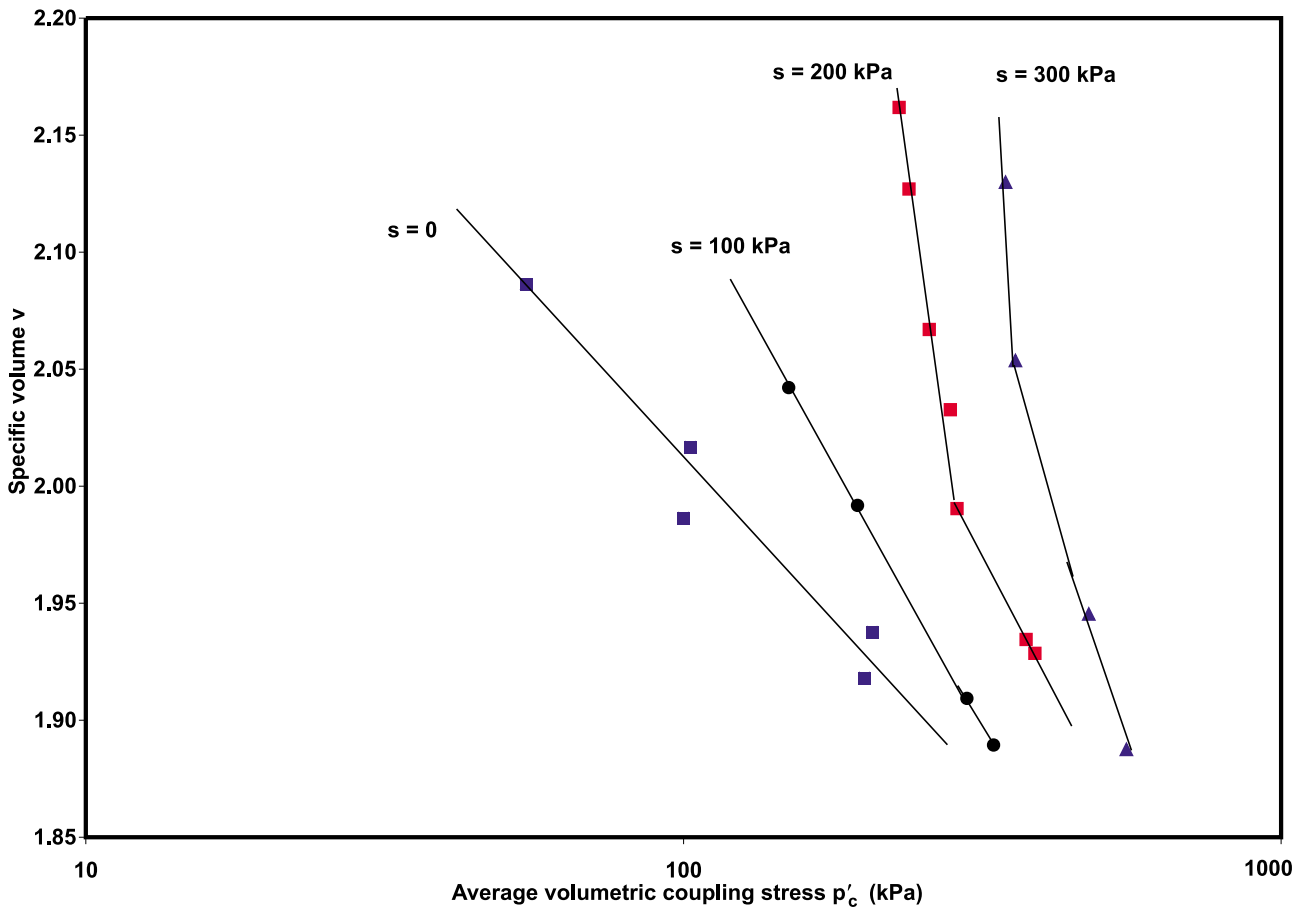


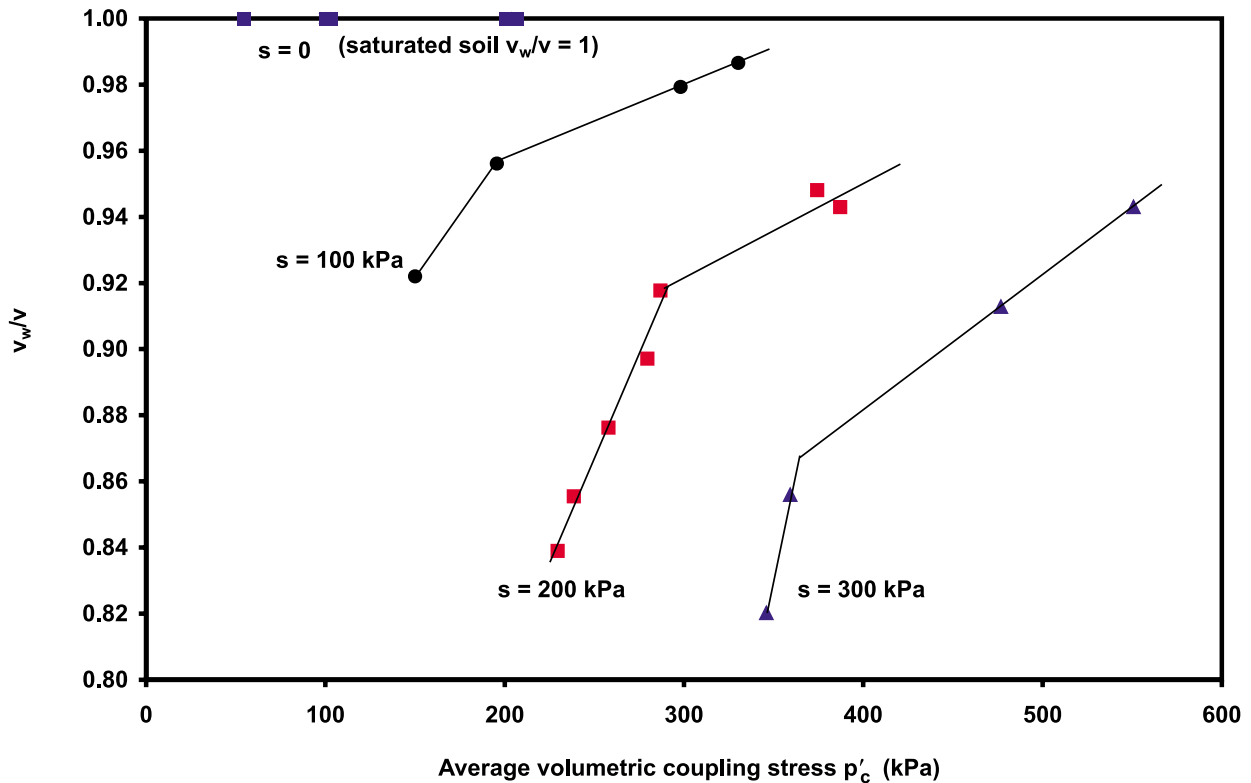
Fig. 12. Plots of specific volume v versus p'_c at critical states from the data of Wheeler and Sivakumar (1995).



a consistent soil structural response to changing applied pressures. The values of the suction stress ratio M_s based on eq. [18], taking $\Lambda = 0.6$, are plotted versus v_w/v in Fig. 11.

The decrease in M_s with decreasing v_w/v is linear, consistent with eq. [18] and with the water phase being drawn back into the finer pores within the soil packets and having a re-

Fig. 13. Plots of v_w/v versus p'_c at critical states from the data of Wheeler and Sivakumar (1995).



ducing influence on the interpacket contacts where shearing is taking place (Toll 1990). As saturated conditions are approached, eq. [18] and Fig. 11 indicate $M_s = \Lambda$ is approached.

Volumetric relations at the critical state

Figure 12 presents the results of v plotted versus p'_c at the critical state, following shearing, from the results of Wheeler and Sivakumar (1995). In plotting $(p - u_a)$ as the coordinate axis, the authors reported the anomalous position of the critical state line for $s = 0$ as it lay between the lines for $s = 100$ kPa and $s = 200$ kPa. Wheeler and Sivakumar also commented that the shapes of the critical state lines were not consistent with the model of Alonso et al. (1990). The use of p'_c in Fig. 12 results in more consistent and expected behaviour trends with no apparent anomaly in the data and with the critical state lines for $s = 100$, 200, and 300 kPa converging on the critical state line for $s = 0$ as p'_c increases. Similarly, the plots of v_w/v versus p'_c (based on the parameters in eq. [15b]) in Fig. 13 show expected trends, with the unsaturated soil specimens tending toward the saturation line as p'_c increases. Of note is the inflection in the plots for unsaturated conditions, suggesting a change in behaviour as saturation is approached. This change occurs at $v \approx 2.0$ for all the suction curves (i.e., when the volume of voids equals the volume of solids). Figure 8 includes normalized plots of q/s and p'_c/s versus v_w/v at the critical state. The plots closely follow single curved lines, implying that the critical state results for unsaturated soils lie on unique curved surfaces in three-dimensional stress–volume space.

Conclusions

Based on considerations of enthalpy, an equation of state for unsaturated soils has been developed. This illustrates (i) the significance of the average volumetric coupling stress (p'_c) in unsaturated soils; (ii) that there is an implicit relationship between the stress state variables in unsaturated soils and the volumes of the phases; and (iii) that a reduction in the suction component $s(v_w/v)$ may help to explain the collapse of sensitive soil structures. The average volumetric coupling stress, p'_c may be viewed as comprising two components, namely $(p - u_a)$ acting throughout the whole soil mass, with an additional stress, s , acting within the soil packets.

Utilizing the data of Wheeler and Sivakumar (1995) for kaolin and the results of Toll (1990) for a lateritic gravel, the equation of state is used to show the following:

(1) When p'_c is plotted as the “controlling” stress during the consolidation and shearing stages of triaxial stress tests, the results follow consistent and logical trends and reported anomalies are clarified.

(2) The shear strength of unsaturated soils is a defined function of the volumes of the phases, and the difference in shear behaviour between saturated and unsaturated soils may be explained in terms of the development, under moisture-deficient conditions, of a bimodal structure comprising saturated, agglomerated-particle packets which act as large particles and are surrounded by air-filled voids.

(3) Normalizing q and p'_c with respect to $s = (u_a - u_w)$ helps in an understanding of the volumetric influence on soil

strength. When plotted against v_w/v , the critical state results for unsaturated soils appear to lie on unique curved surfaces. Normalizing also helps to unify consolidation data.

Constitutive modelling using independent stress state variables is established in research into the behaviour of unsaturated soils. Equation [15] provides a link between the stress state variables and the volumes of the phases in a soil and assists in providing a readily understood interpretation of experimental data.

Acknowledgment

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List of symbols

- a dimensionless number in eq. [7]
 A enthalpy component associated with the adsorbed water
 A_c shape factor for contractile skin of a saturated packet
 b dimensionless number in eq. [11]
 C enthalpy component associated with the contractile skin
 D enthalpy component associated with the dissolved air
 e voids ratio
 G_s specific gravity of the soil particles
 H_a enthalpy of the air phase
 H_n enthalpy of a phase
 H_s enthalpy of the solid phase
 H_t total enthalpy
 H_w enthalpy of the water phase
 m ratio of pore-water pressure change to mean total stress change
 M critical state stress ratio
 M_s suction stress ratio
 M_t total stress ratio
 n_a air porosity ($= V_a/V$)
 n_s V_s/V
 n_w water porosity ($= V_w/V$)
 N number of saturated packets per unit volume of soil
 N_t extrapolated value of v at $p_c' = 1.0$ kPa
 p mean total stress
 p' Terzaghi's mean effective stress
 p_B' Bishop's mean stress
 p_c' average volumetric "coupling" stress
 p_f' mean "effective" stress within a saturated packet due to suction
 p_n pressure in a phase
 q deviator stress
 R radius of spherical occluded air bubbles
 R_c radius of air bubble under equilibrium conditions
 R_1, R_2 radii of curvature of the contractile skin for two perpendicular planes at right angles to the curvature of the interface
 R^* radius of a spherical water droplet in air
 s suction ($= u_a - u_w$)
 S_r degree of saturation
 T_c surface tension in the contractile skin
 u_a pore-air pressure
 u_s stress in the soil particles
 u_v water vapour pressure
 u_w pore-water pressure
 u_w^* water pressure in a droplet of radius R^*
 U_n internal energy associated with phase interactions
 v specific volume ($= 1 + e$)

- v_w specific water volume ($= 1 + eS_r = 1 + wG_s$)
 V total volume
 V_a volume of the air phase
 V_n volume of a phase
 V_p volume of saturated packet
 V_s volume of solid phase
 V_w volume of water phase
 W enthalpy components associated with the water vapour
 w water content
 α dimensionless parameter defined by eqs. [12] and [13]
 λ_t critical state deformation parameter
 χ empirical parameter in Bishop's eq. [2]
 Λ intercept on the q/s axis at $p'_c/s = 1$

Subscripts:

i, f initial and final conditions

Appendix A. Determination of b in eq. [10]

The following examines the enthalpy and thus the stresses within saturated packets of soil. This allows an assessment of the component of enthalpy within the contractile skin of an unsaturated soil. Consider the case of a spherical water droplet of radius R^* in air. The water pressure within such a droplet would be greater than the surrounding air pressure. If small soil particles were added to the droplet, the overall volume of water and particles combined would increase and the water pressure would decrease as the overall radius of the droplet increased. Eventually a situation would be reached where the volume of soil particles would result in water pressure less than the surrounding air pressure and in this situation a saturated packet would be created. The sum of the individual enthalpies of the free water droplet (including that of the contractile skin, noting that the water pressure exceeds the air pressure) and the particles before they were combined is equated to the total enthalpy of the saturated packet (including that of the effective stress and the contractile skin, noting that now the air pressure exceeds the water pressure) after they are combined. The assumption is made that the volumes of water and soil particles remain unchanged and thus no work is done under isothermal conditions. Ignoring the small effect of the water vapour pressure and using the subscripts i and f to denote initial and final conditions, respectively,

$$[A1a] \quad H_{wi} + H_{si} = H_{wf} + H_{sf}$$

or

$$[A1b] \quad u_w^* V_w - C_i + u_{si} V_s = u_w V_w + C_f + u_{sf} V_s + p'_f V$$

where

$$\begin{aligned}
 H_{wi} &= u_w^* V_w - C_i = u_w^* V_w - \frac{3}{2} (u_a - u_w^*) V_w; \\
 H_{si} &= u_{si} V_s = u_a V_s; \\
 H_{wf} &= u_w V_w + C_f = u_w V_w + A_c (u_a - u_w) V_w; \\
 H_{sf} &= u_{sf} V_s + p'_f V = au_w V_s + p'_f (V_w + V_s); \text{ and} \\
 u_w^* &\text{ is the water pressure in a droplet of radius } R^*.
 \end{aligned}$$

$$[A2] \quad u_a - u_w^* = -\frac{2T_c}{R^*}$$

where

C_i is the component of enthalpy of contractile skin of a spherical water droplet (determined in a similar way to eq. [9] for air bubbles);

C_f is a component of enthalpy of contractile skin in a saturated packet;

A_c is a shape factor for the contractile skin of a saturated packet;

p'_f is the mean effective stress within a saturated packet due to suction alone; and

$$V = V_w + V_s.$$

For a number of isolated saturated packets, eq. [A1b] can be written as

$$\begin{aligned}
 [A3] \quad -\frac{3}{2} (u_a - u_w^*) n_w + u_w^* n_w + u_a n_s &= A_c (u_a - u_w) n_w \\
 &+ u_w n_w + au_w n_s + p'_f (n_w + n_s)
 \end{aligned}$$

where

$$[A4] \quad n_w = N \frac{4\pi}{3} (R^*)^3$$

where N is the number of saturated packets per unit volume of soil (the total volume of water in the saturated packets must equate to the volume of water contained in the free water droplets).

Within the saturated regions, Terzaghi's effective stress is considered to prevail and $p'_f = (p - u_w)$ applies. The stress acting external to the isolated saturated regions created as in eq. [A3] is u_a , thus $p'_f = (u_a - u_w)$. Substituting for p'_f in eq. [A3] gives

$$[A5] \quad A_c (u_a - u_w) n_w = b (u_a - u_w) (n_w + n_s)$$

where

$$b = \frac{n_w}{n_w + n_s} \left[1 - \frac{5(u_a - u_w^*)}{2(u_a - u_w)} + \frac{(u_a - au_w)n_s}{(u_a - u_w)n_w} \right] - 1$$

Appendix B. Assessment of the values of parameters a , b , and α

It is necessary to assign a size to the saturated packets. These are assumed to vary between silt and gravel size (0.01–1.00 mm) and, for simplicity, to be spherical, although this is not considered to detract from the conclusions reached. The smaller the packet, the greater the influence on α . Silt size has been chosen as the lower limit because it has been noted that silt-size agglomerations are present in clays that have been subjected to repeated extremes of wetting and drying (e.g., Chandler and Davis 1973). The upper limit has been chosen as gravel size, although packets are likely to be considerably larger than this in soils where there is a low air voids content.

Example calculations

From a selected test result of Wheeler and Sivakumar (1995), at the critical state, $n_a = 0.087$, $n_w = 0.399$, $n_s = 0.514$, $u_a = 349.75$ kPa, $u_w = 50.36$ kPa, and $(u_a - u_w) = 299.39$ kPa.

For a silt-size saturated packet, the volume V_p of a saturated packet = $5.24 \times 10^{-13} \text{ m}^3$ and $N = (n_w + n_s)/V_p = 1.74 \times 10^{12}/\text{m}^3$. From eq. [A4], the radius of a free water droplet $R^* = 3.79 \times 10^{-5} \text{ m}$. From eq. [A2], taking $T_c = 73 \times 10^{-6} \text{ kN/m}$ at 20°C , $(u_a - u_w^*) = -3.85 \text{ kPa}$. From eq. [14], $\alpha = 0.986$.

For a gravel-size packet, $V_p = 4.19 \times 10^{-9} \text{ m}^3$, $N = 2.18 \times 10^4/\text{m}^3$, $R^* = 7.59 \times 10^{-4} \text{ m}$, $(u_a - u_w^*) = -0.19 \text{ kPa}$, and $\alpha = 0.999$. There is little error in the assumption of $\alpha = 1$.

Equation [A5] indicates that b as given by eq. [10] is a maximum (and thus the enthalpy due to surface tension is a maximum) when a is at its minimum of 1. The term a is defined by eq. [7] for a saturated packet. Thus, since b has a minimum value of 0, a range of values of a and b for the sat-

urated packets can be determined. For a silt-size packet, $1 \leq a \leq 1.15$ and $0.14 \geq b \geq 0$; for a gravel-size packet, $1 \leq a \leq 1.01$ and $0.0007 \geq b \geq 0$.

The foregoing indicates that the component of enthalpy in the soil particles as a result of imposed air and water pressures under unsaturated conditions (eq. [7]) can be taken as that under saturated conditions ($u_w V_s$) without any appreciable error, and that the component of enthalpy due to surface tension in the contractile skin (eq. [10]) is small and can be ignored. The values of a and b thus determined might have been judged intuitively as reasonable assumptions. This leads to $\alpha = 1$ being a sensible practical interpretation for the unsaturated soil tests considered.